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Equations of Motion for Tether-connected Dumb-bell Satellites System under Various Perturbations of General Nature

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Abstract:

The work represents the equations of motion for a system of two tether-connected artificial satellites under the influence of several perturbations of general nature. The set of equations is non-linear, non-homogeneous and non-autonomous second order differential equations. In our problem, the tether is taken as non-conducting, light, flexible and inelastic in nature. We have treated the problem by taking six perturbative forces on the system simultaneously. Out of six perturbations, three perturbations exist due to influence of the earth, namely geo-magnetic field, shadow and oblateness. The other three perturbations are due to the air drag, solar light pressure and wobbling of the orbit of centre of mass of the system. The motion of the system is studied relative to its centre of mass. It is considered that the centre of mass of the system moves in a Keplerian elliptical orbit.

Key-words: Equations of motion, Inelastic tether, Nechvile's co-ordinate system, Perturbative forces of general nature, Rotating frame of reference.

Introduction:

The present work was initiated and founded by Beletsky [1] studying the relative motion of two cable- connected bodies moving in orbit to centre of mass Further, the work of a cable connected two satellites systems in the field of central gravitational force was done by two Russians researchers Okhostimsky and Sarichev [2]. They investigated the motion of an artificial satellite attached with two material particles with the help of rigid rods. Later, the problem was elaborated by Sarichev [3] considering the effect of various perturbative forces on the stability of the system under gravity. Chobotov [4] studied the motion of a dumb-bell satellite spinning in the orbit and gravity - gradient excitation. Beletsky, Novikova and Novoorebelskii [5-6] are the pioneer workers in researches related to a cable-connected satellites system. They initiated to work on the motion of satellites under gravitational force of the earth. The two satellites are connected by a light and non-conducting tether. The tether is flexible and inelastic in nature also. It was considered that the motion of the above satellites system takes place only in circular orbit about its centre of mass. Bhattacharya et al. [7] studied about stability of a system of two cable-connected satellites in earth's magnetic field. Later, Kumar and Bhattacharya [8-9] investigated the influence of earth's magnetic field, solar radiation pressure and earth's oblateness on the stability during motion of two cable-connected satellites system. Prasad and Kumar [10-11] handled the problem by taking an account of different aspects of the analytical simulation and physics of non - linearity. Again, Kumar and Kumar [12] investigated about equilibrium positions of a tether - connected system in the presence of solar radiation pressure, shadow of the earth, earth's oblateness and air resistance in circular orbit. Study of dynamics of a tethered satellite formation for space exploration and coupling dynamics characteristics of simplified model for tethered satellite

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system was performed by various researchers [13-15]. Kumar and Ghosh [16] investigated equations of motion in elliptical orbit in case of elastic-connected satellites under various perturbations. Several authors [18-20] could study the non-linear dynamics of artificial satellites attached through a tether under the combined effect of different perturbating forces for obtaining stable positions of the system.

2. Equation of motion for the system: -

According to the first principle of mechanics, we may write the Lagrange's Equations of motion for the system of dumb-bell satellites as [17]

$$m_1\ddot{\vec{r_1}} = -\mu\frac{m_1\vec{r_1}}{r_1^3} + \lambda(\vec{r_1} - \vec{r_2}) + Q_1\left(\dot{\vec{r_1}} \times \vec{H}\right) + \gamma B_1\vec{n_1} - 3\frac{m_1\mu k_2}{r_1^5}\vec{r_1} - \rho_a c_1 m_1|\dot{\vec{r_1}}|\dot{\vec{r_1}} + \sqrt{\mu/r_1^3} \tag{1}$$

And

$$m_2\ddot{\vec{r_2}} = -\mu\frac{m_2\vec{r_2}}{r_2^3} + \lambda(\vec{r_2} - \vec{r_1}) + Q_2\left(\dot{\vec{r_2}} \times \vec{H}\right) + \gamma B_2\vec{n_2} - 3\frac{m_2\mu k_2}{r_2^5}\vec{r_2} - \rho_a c_2 m_2|\dot{\vec{r_2}}|\dot{\vec{r_2}} + \sqrt{\mu/r_2^3} \tag{2}$$

Where μ= GM= Product of gravitational constant and mass of the earth so it will be a constant quantity.

Also

$$r_1 = R_E + h_1\,, \quad r_2 = R_E + h_2$$

Here

h_1 = Altitude of the orbit for the first satellite above the earth surface.

h_2= Altitude of the orbit for the second satellite above the earth surface.

In the present case the condition of constraint are as follows

$$|\vec{r_1} - \vec{r_2}| \le l \tag{3}$$

If the inequality sign holds well in (3), then the system moves without any constraint. This is called "Free motion" of the system. If equality sign holds well, then the system moves with the active constraint. This is called "Constrained motion". But in practice, motion of the system is a combination of free and constrained motion.

Here, the masses of the two satellites are denoted by m_1 and m_2. B_i (i = 1, 2) is the absolute values of the forces due to the direct solar radiation pressure on m_i (i = 1, 2) and are small. Q_1 & Q_2 are the charges of the two satellites. μ_E is the magnitude of magnetic moment of the earth's dipole. p represents the focal parameter. λ is undetermined Lagrange's multiplier. e is eccentricity of the orbit of the centre of mass. v represents the true anomaly.

$\in$ represents inclination of the oscillatory plane. α denotes ray's inclination. γ is a shadow function which depends on the illumination of the system of satellites by the sun rays. If γ is equal to zero, then the system is affected by the shadow of the earth. When the motion is under the influence of direct solar pressure, the value of γ is one. R indicates the position vector for C.M. in magnitude. c_1 and c_2 are the Ballistic coefficients. ρ_a is the average density of the atmosphere. i is inclination of the orbit with the equatorial plane.

The dash ($'$) indicates differentiation with respect to v. $\bar{r}_1$ and $\bar{r}_2$ are the radius vectors of the particles m_1 and m_2 respectively. $\bar{H}$ is the intensity of earth's magnetic field. $\bar{n}_1$ and $\bar{n}_2$ are the unit vectors in the direction of the sun rays towards m_1 and m_2 respectively. The length of the tether connecting m_1 and m_2 is represented by ℓ.

As, we see that h_1 and h_2 are approximately equal, therefore we can simply write r in place

of r_1 and r_2 . Again, $\overrightarrow{n_1}$ and $\overrightarrow{n_2}$ are almost parallel, so we can replace them by $\vec{n}$
Hence, by adding the equations (1) and (2), we find

$$m\ddot{\vec{R}} = -\mu\left[\frac{m_1(\overrightarrow{R_1}+\overrightarrow{\rho_1})}{(R+\rho_1)^3}+\frac{m_2(\vec{R}+\overrightarrow{\rho_2})}{(R+\rho_2)^3}\right]+\left(Q_1\dot{\vec{r}}_1+Q_2\dot{\vec{r}}_2\right)\times H+\gamma(B_1+B_2)\vec{n}-3\mu k_2\left[\frac{m_1(\vec{R}+\overrightarrow{\rho_1})}{(R+\rho_1)^5}+\frac{m_2(\vec{R}+\overrightarrow{\rho_2})}{(R+\rho_2)^5}\right]-\rho_a\left[c_1m_1\left|\left(\dot{\overrightarrow{R_1}}+\dot{\overrightarrow{\rho_1}}\right)\right|\left(\dot{\vec{R}}+\dot{\overrightarrow{\rho_1}}\right)+c_2m_2\left|\left(\overrightarrow{R_1}+\dot{\overrightarrow{\rho_2}}\right)\right|\left(\dot{\vec{R}}+\dot{\overrightarrow{\rho_2}}\right)\right]$$
$$+2\sqrt{\mu/r^3}$$
(4)
Here,

$m = (m_1+m_2)$, total mass of the system

And

$\vec{R} = \left(\frac{m_1\overrightarrow{r_1}+m_2\overrightarrow{r_2}}{m_1+m_2}\right)$radius vector of C. M. of the system w.r.t. the origin of the attracting centre
Also

$\vec{r}_j,\ \vec{R}\ and\ \vec{\rho}_j$ (j=1,2) are related as

$$\vec{r}_j = \vec{R}+\vec{\rho}_j \quad \text{(j=1,2)} \tag{5}$$

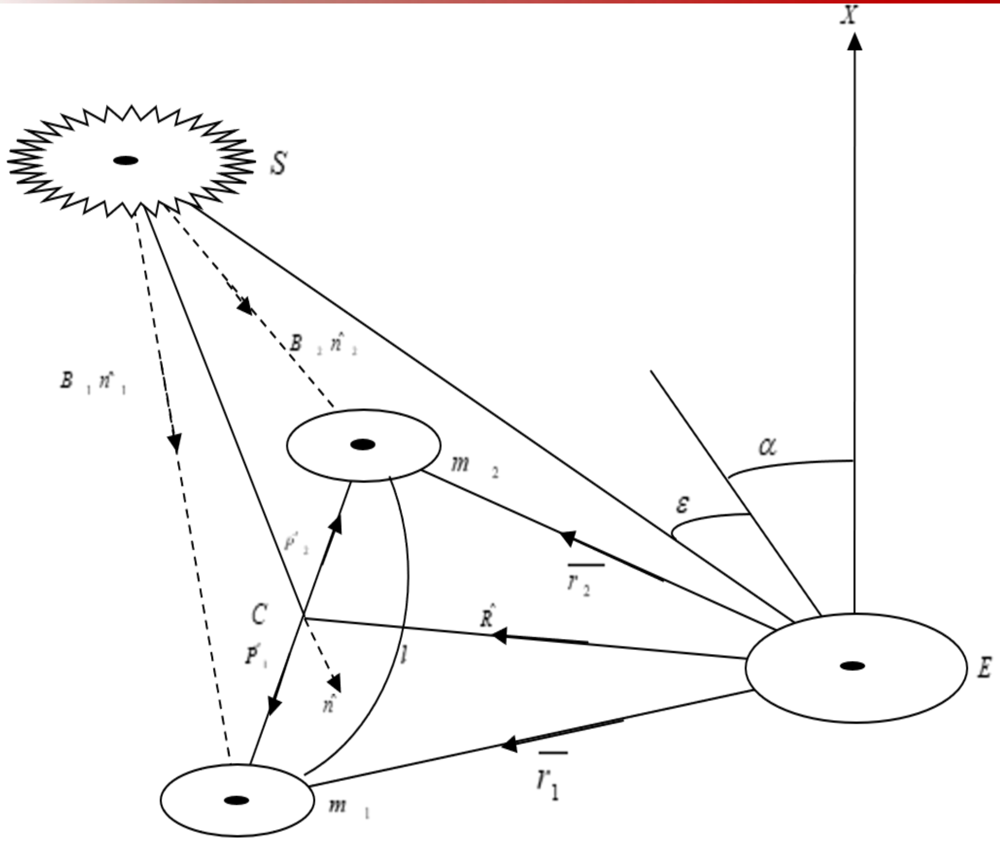

Figure 1: diagrammatical representation of the tether-connected dumb-bell satellite system under various perturbations.

Further,

The radius vector $\vec{\rho}_j$ of the particle m_j w.r.t. the center of mass of the system C as shown in figure (1) shall be governed by a relation

$$m_1\overrightarrow{\rho_1} + m_1\overrightarrow{\rho_2} = 0 \quad (6)$$

It is observed that C.M. of the system is the origin for the frame of reference. Thus, the relation (6) is fulfilled.

Now,

Expanding equation (4) in power of $\frac{\rho_j}{R}$ upto the first order of infinitesimals, we get

$$m\ddot{\vec{R}} + \frac{\mu m\vec{R}}{R^3} = \overrightarrow{F_1} + \overrightarrow{F_2} + (Q_1 + Q_2)\left(\dot{\vec{R}} \times H\right) + \gamma(B_1 + B_2)\vec{n} - 3\mu k_2\left[\frac{m_1(\vec{R}+\overrightarrow{\rho_1})}{(R+\rho_1)^5} + \frac{m_2(\vec{R}+\vec{\rho}_2)}{(R+\rho_2)^5}\right] - a\dot{\vec{R}} + 2\sqrt{{}^{\mu}/_{r^3}} \quad (7)$$

Where

$$\vec{F_1} = \frac{3\mu}{R^3}(m_1\vec{\rho}_1 + m_2\vec{\rho}_2) + \frac{3\mu}{R^5}[\vec{R}\,(m_1\vec{\rho}_1 + m_2\vec{\rho}_2)]\vec{R}$$

$$\vec{F_2} = \frac{3\mu}{2R^3}\left|m_1\left\{\left(\frac{\rho_1}{R}\right)^2 - 5\left(\frac{\vec{R}\,\vec{\rho_1}}{R\;R}\right)^2\right\} + m_2\left\{\left(\frac{\rho_2}{R}\right)^2 - 5\left(\frac{\vec{R}\,\vec{\rho_2}}{R\;R}\right)^2\right\}\vec{R}\right| + \frac{3\mu m_1}{R^5}(\vec{R}.\vec{\rho_1})\vec{\rho_1} + \frac{3\mu m_2}{R^5}(\vec{R}.\vec{\rho_2})\vec{\rho}_2$$

And

$$a_1 = \rho_a R(c_2 - c_1)\left(\frac{m_1}{m_1+m_2}\right). \qquad (8)$$

By using relation (6) in equation (8), we find that

$$\vec{F_1} = 0 \qquad (9)$$

Again,

$$\text{As } \rho_j << r_j,\; r_j \sim R \;\text{ and }\; \frac{\rho_j}{R} << 1$$

$$\vec{F}_2 = 0 \qquad (10)$$

Substituting equations (9) and (10) in equation (7) and neglecting the terms of k_2, we shall get

$$m\ddot{\vec{R}} + \frac{\mu m\vec{R}}{R^3} + (Q_1 + Q_2)\left(\dot{\vec{R}} \times H\right) - \gamma(B_1 + B_2)\vec{n} + a_1\dot{\vec{R}} + 2\sqrt{{}^{\mu}/_{r^3}}^{\,\cdot} = 0 \qquad (11)$$

Again, the effect of air resistance on the motion of the system may be neglected as it is of perturbative nature. Therefore, we can write (11) as

$$m\ddot{\vec{R}} + \frac{\mu m\vec{R}}{R^3} + (Q_1 + Q_2)\left(\dot{\vec{R}} \times H\right) - \gamma(B_1 + B_2)\vec{n} + 2\sqrt{{}^{\mu}/_{r^3}} = 0 \qquad (12)$$

Now, taking scalar product of both the sides of equation (12) with $\dot{\vec{R}}$, we get

$$m\ddot{\vec{R}}\dot{\vec{R}} + \frac{\mu m\vec{R}\dot{\vec{R}}}{R^3} + 2\sqrt{{}^{\mu}/_{r^3}} = 0 \qquad (13)$$

$$\vec{n}.\dot{\vec{R}} = 0 \qquad (14)$$

Equation (14) represents usual case for satellites of the earth.
Finally, integrating (13), we obtain`

$$\dot{\vec{R}}^2 = A + \frac{2\mu}{R} + 2\sqrt{{}^{\mu}/_{r^3}} \qquad (15)$$

Where, A is a constant of Integration having dimension of energy per unit of mass. Equation (15) states that C. M. of the system describes an elliptical orbit. Usually, the above relation is the energy equation of the central force under inverse square law.

3. Result and Discussion:

In the present research paper, the equation of energy for centre of mass of the system has been obtained starting from the Lagrangian equations of motion of the first kind for individual satellite under the influence of earth's shadow, solar pressure, air drag, earth oblateness, geo-magnetic field and wobbling of the orbit of centre of mass of the system. The gravitational force of the earth is the main force and all the above six forces due to their small magnitude are taken as perturbative forces. The centre of mass of the system has been assumed to move along a Keplerian orbit. The obtained total energy is negative and orbit is elliptical in nature.

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