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First Integral of Motion of a Tether Connected Satellites System Under Several Influences of General Nature: Circular Orbit

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Abstract:

Here, we are interested to obtain Jacobean integral of motion of a system of two elastic cable connected satellites in circular orbit under the influence of some perturbative forces of general nature. These perturbative forces are Geo-magnetic field, solar radiation pressure, shadow of the earth and air drag. We also consider elasticity of the cable as one of the perturbative forces i.e., elasticity of the cable also influences motion of the system as a whole. Shadow of the earth is taken to be cylindrical and the entire system is allowed to pass through the shadow beam. Off course, gravitational force of the earth is the main force that governs motion of the system. The cable connecting the two satellites is taken to be light flexible, non-conducting and elastic in nature. Since, masses of the satellites are small and distance between the satellites is very large, the gravitational forces of attraction between the two satellites and other celestial bodies including the sun have been neglected. We do not consider nutation and wobbling of the orbit.

Keywords: Jacobean Integral, Elastic Cable, Tether Connected Satellites, perturbative forces of general nature, Circular Orbit

Introduction:

In the beginning, the motion of the two tether connected satellites system was studied by Beletsky and Novikova (1969). They performed their research work in the gravitational field of the earth. The two workers assumed in the work that the centre of mass of the two satellites moves in a circular orbit. Singh and Demin (1972) and Singh (1973) elaborated the problem in two dimensional and three-dimensional cases as well. The influence of magnetic force on a cable-connected satellites system was studied by Das et al. (1976). The influence of solar radiation pressure on the motion and stability of a cable-connected satellites system was investigated by Sinha and Singh (1987). Development of satellite technology and its impact on social life was carried out by Umar (2013). Kumar and Prasad (2015) worked on non-linear planer oscillations of a cable-connected satellites system and non-resonance. Liberation points of a cable-connected satellites system under several influences was studied by Kumar and Kumar (2016).

The present research work is an idealisation of real space system. It is based on well known physical and mathematical analysis and simulation techniques. In-fact, the work is a restricted three body problem. Jacobean integral of motion of the system of two tether connected artificial satellites is obtained under the influence of several perturbative forces. These perturbative forces are Earth's magnetic field, Solar radiation pressure, shadow of the earth, air-drag and elasticity of the cable. The influence of the above mentioned perturbations on the system has been studied singly and by a combination of any two or three or four of them by various workers, but never con-jointly all at a time. Therefore, the work of previous workers could not give a real picture of motion of the system concerned system. This fact has initiated the present

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research work. Gravitational force of the earth is the main force and all other forces are considered here as perturbing forces.

Treatment of the Problem:

Equations of motion of one of the satellites when the centre of mass moves along Keplerian elliptical orbit is written as-

$$X'' - 2Y' - 3X\rho = -\frac{A}{\rho}\cos i - \gamma\left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right)\cos\in\cos(v-\alpha) - f\rho\rho'$$

$$-\lambda_\alpha\left[\rho - l_0\left(X^2+Y^2+Z^2\right)^{-1/2}\right]X$$

$$Y'' + 2X' = -\frac{A\rho'}{\rho^2}\cos i + \gamma\left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right)\cos\in\sin(v-\alpha) - f\rho^2$$

$$-\lambda_\alpha\left[\rho - l_0\left(X^2+Y^2+Z^2\right)^{-1/2}\right]Y \qquad (1)$$

The condition of constraint is-

$$X^2 + Y^2 \le \frac{l_0^{\,2}}{\rho^2} \qquad (2)$$

Also,

$$\rho = \frac{1}{1+e\cos v}, \quad \lambda_\alpha = \left[\frac{m_1+m_2}{m_1 m_2}\right]\frac{\lambda}{l_0}, \; k_2 = \frac{\bar{\varepsilon}\,\mathrm{R_e}^2}{3}, \; \bar{\varepsilon} = \alpha_R - \frac{m}{2}, \; m = \frac{\Omega^2\,\mathrm{R_e}}{g_e}$$

$$A = \left(\frac{m_1}{m_1+m_2}\right)\left(\frac{Q_1}{m_1} - \frac{Q_2}{m_2}\right)\frac{\mu_E}{\sqrt{\mu p}} \quad , \quad f = \frac{a_1 P^3}{\sqrt{\mu\rho}} \qquad (3)$$

μ= product of mass of the earth and gravitational constant.

P is focal parameter, m_1 & m_2 are the masses of the first and second satellite respectively, B_1 & B_2 are the absolute values of the forces due to the direct solar pressure on m_1 & m_2 respectively, Q_1 & Q_2 are the charges of the two satellites, μ_E is the magnitude of magnetic moment of the earth's dipole, λ = Lagrange's multiplier, g_e=force of gravity, e= eccentricity of the orbit of the centre of mass, v =true anomaly of the centre of mass of the system, ϵ is inclination of the oscillatory plane, α is the inclination of the ray , γ is a shadow function. If the system is affected by the shadow of the earth, then γ is equal to zero. On the other hand, if the system is not within the said shadow, then γ is equal to one.c_1 and c_2 are the Ballistics coefficients, R is the modulus of position vector , ρ_a is the average density of the atmosphere,

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i is inclination of the orbit with the equatorial plane, Θ_2 is the angle between the axis of the cylindrical shadow beam and the line joining the centre of the earth and the end point of the orbit of the centre of mass within the earth's shadow, prime denotes differentiation with respect to v.

If the motion of one of the satellites be determined with the help of equation (1) then motion of the other satellite of mass m_2 can be determined by Kumar & Kumar [10]-

$$m_1\bar{\rho}_1 + m_2\bar{\rho}_2 = 0 \qquad (4)$$

In the case of circular orbit of centre of mass of the system, we must write e = 0. Therefore, $\rho = 1$ and $\rho' = 0$. In this way we shall write equations (1) as-

$$X''-2Y'-3X = -A\cos i - \gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\epsilon\cos(v-\alpha) - \lambda_\alpha[1-l_0(X^2+Y^2)^{-1/2}]X$$

$$Y'' + 2X' = \gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\epsilon\sin(v-\alpha) - f - \lambda_\alpha[-l_0(X^2+Y^2)^{-1/2}]Y \qquad (5)$$

Condition of constraint is given by – $X^2 + Y^2 \le l_0^2$

The system of two satellites is allowed to pass through the shadow beam during its motion. Let us assume that Θ_2 is the angle between the axis of the cylindrical shadow beam and the line joining the centre of the earth and the end point of the orbit of the centre of mass within the earth's shadow, considering the direction towards the motion of the system. The system starts to be influenced by the solar pressure when it makes an angle Θ_2 with the axis of the shadow beam remains under the influence of solar radiation pressure till it makes an angle (2π- Θ_2) with the axis of the cylindrical shadow beam. Thereafter, the system will enter the shadow beam and the effect of solar pressure will come to an end.

Next, the periodic terms can be averaged to assess the long-term in (5) and secular effects of solar pressure as well as the effects of the earth's shadow on the system.
$$X'' -2Y'-3X = -A\cos i +\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\epsilon\sin\alpha\sin\Theta_2/\pi - \lambda_\alpha[1-l_0(X^2+Y^2)^{-1/2}]X$$

&

$$Y'' + 2X' = \left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\epsilon\sin\alpha\sin\Theta_2/\pi - f - \lambda_\alpha[1-l_0(X^2+Y^2)^{-1/2}]Y \quad (7)$$

Since, the above equation do not contain the time explicity, Jacobean integral of motion of the system must exist.

To obtain Jacobean Integral of motion of the system we multiply the 1st and 2nd equation of (7) by 2X' and 2Y' respectively and we get-

$$2X'(X''-2Y'-3X)=2X'[-A\cos I +\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\epsilon\sin\alpha\sin\Theta_2/\pi - \lambda_\alpha[1-l_0(X^2+Y^2)^{-1/2}]X]$$

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$2X'X'' - 4X'Y' - 6XX' = -2x'A\cos i + 2X' \left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right)\cos \epsilon \sin \alpha \sin \Theta_2 / \pi - 2X' \lambda_\alpha [1 - l_0 (X^2 + Y^2)^{-1/2}]X$

$2Y'(Y'' + 2X') = 2Y'\left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right) \cos \epsilon \sin \alpha \sin \Theta_2 / \pi - 2Y'f - 2Y'(\lambda_\alpha[1 - l_0 (X^2 + Y^2)^{-1/2}]Y)$

$2Y'Y'' + 2Y' 2X' = 2Y'\left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right) \cos \epsilon \sin \alpha \sin \Theta_2 / \pi - 2Y' f - 2Y'\lambda_\alpha[1 - l_0(X^2 + Y^2)^{-1/2}]Y$

Adding and then integrating we get-

$$X'^2 + Y'^2 - 3X^2 = -2A X \cos i + 2/\pi \left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right) \cos \epsilon \sin \Theta_2 (X \cos \alpha + Y \sin \alpha) - 2fY - \lambda_\alpha[(X^2 + Y^2) - 2l_0 (X^2 + Y^2)^{1/2}] + h \quad (8)$$

Next, we examine the surface of zero velocity of motion of the system. For this purpose, we take help from the Jacobean integral (8), It is as follows------

$$3X^2 - 2AX \cos i + 2/\pi \left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right)\cos \epsilon \sin \Theta_2 (X \cos \alpha + Y \sin \alpha) - 2fY$$

$$-\lambda_\alpha[(X^2 + Y^2) - 2l_0 (X^2 + Y^2)^{1/2}] + h = 0 \quad (9)$$

Results and Discussions –

The purpose of the present research work is to obtain Jacobean integral of motion of a system of two cable connected satellites under the influence of several perturbative forces of general nature.The cable is taken as light, flexible, non-conducting and elastic in nature. During the motion, the satellites cut the magnetic lines of force of the earth and hence charges get developed on the two satellites. The magnitude of the developed charge on the two satellites are very small. Hence, electrostatic force between the the two satellites is not taken into account.

Conclusion:

The above obtained equation i.e., equation (9) is the required Jacobean Integral of motion of the system. The equation we obtained in (9) has a very wide applications in the further studies of two elastic cable connected artificial satellites system.

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